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ANALYSIS OF PRIMES IN ARITHMETICAL PROGRESSIONS $4n + k$ UP TO A TRILLION

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ABSTRACT

An in-depth analysis of the highly irregularly distributed prime numbers of the form $4n + k$ in the range of 1 to 1 trillion is presented in this work. Since we widely use the decimal number system, the distribution trends of the primes in the blocks of all powers of 10 are explicitly explored. This is expected to throw some light on the sought hitherto unknown pattern of prime distribution.

KEYWORDS: Prime Numbers, Arithmetical Progressions, Prime Density, Block-wise Distribution, Prime Spacing

MATHEMATICS SUBJECT CLASSIFICATION 2010:11A41, 11N05, 11N25

INTRODUCTION

Every prime number is an integer greater than 1 which has only two positive divisors, viz., 1 and itself. That the number of primes is infinite is known from time of Euclid and has various proofs with different interesting approaches [6].

PRIMES DISTRIBUTIONS

Prime numbers seem highly irregularly distributed amongst the positive integers. The number of primes less than or equal to a given positive value x is denoted by a function $\pi(x)$.

There have been many attempts for formulations of prime distribution. It is remains unsettled question: whether there exists a regular pattern of occurrence of primes or not?

Prime Distributions In Arithmetical Progressions

Excluding the first prime 2 all others are odd.All primes except 2 find their place in the arithmetical progression $2n + 1$. This arithmetical progression $2n + 1$ contains infinitely many, in fact all (apart from first one, viz., 2) primes. At the same time, all members of $2n + 1$ are not primes. It contains infinitely many non-primes, i.e., the composite numbers also!

The question of 'whether there are other arithmetical progressions which contain only primes' was easily settled in negation. Dirichlet [1] proved the weaker version of this query '*there are other arithmetical progressions which contain infinitely many primes*'. Dirichlet's Theorem says that an arithmetical progression $an + b$ with $\gcd(a, b) = 1$ contains infinitely many primes.

Since then that there have been consistent efforts to analyze the number of primes occurring in various arithmetical progressions for of getting some direct or indirect hints about prime distribution. The number of primes less than or equal to a given positive value x and that are of the form $an + b$ is denoted by a function $\pi_{a,b}(x)$.

Primes In The Arithmetical Progressions 4*n* + 1 AND 4*n* + 3

Integer division gives one of the numbers $0, 1, 2, \cdots, m - 1$ as remainders after dividing any positive integer by m. We consider $m = 4$ here, so that the possible values of remainders in the process of division by 4 are 0, 1, 2, and 3. Since every positive integer after dividing by 4 has to yield as remainder one and only one amongst these values, it must be of either of the forms $4n + 0 = 4n$ or $4n + 1$ or $4n + 2$ or $4n + 3$, which constitute arithmetical progressions.

First few numbers of the form $4n$ are

$4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, \cdots$

As can be clearly seen, each of these is perfectly divisible by 4. None of these is prime. Thus this sequence contains no prime and its all members are composite numbers. It becomes evident also by seeing $4n$ as arithmetical progression $4n + 0$, where $\gcd(4, 0) = 4 > 1$ and by Dirichilet's Theorem, this is just not a candidate to look ahead for occurrence of many primes.

First few numbers of the form $4n + 1$ are

$1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, \cdots$

This does contain infinitely many primes as $\gcd(4, 1)$ is 1 as per requirement of Dirichlet's Theorem.

First few numbers of the form $4n + 2$ are

$2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, \cdots$

Each of these is even and hence divisible by 2. Except the first member, viz., 2, none of these is prime. Thus this sequence contains only one prime 2 and its all other members are composite numbers. It becomes evident also by seeing $4n + 2$ as arithmetical progression where $\gcd(4, 2) = 2 > 1$ and by Dirichilet's Theorem, this is also just not a candidate to look ahead for occurrence of many primes.

First few numbers of the form $4n + 3$ are

$3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \cdots$

This sequence also contains infinitely many primes as $\gcd(4, 3)$ is 1 as per requirement of Dirichlet's Theorem. There are independent proofs about infinitude of primes of types $4n + 1$ and $4n + 3$ [2, Lemma 3.1.5.1].

We present here a comparative analysis of the primes occurring in arithmetical progressions $4n + k$.

PRIME NUMBER RACE

For a specific positive integer a and all integers b with $0 \leq b < a$, all the arithmetical progressions $an + b$which contain infinitely many primes are compared to decide which one amongst them contains more number of primes. This is term popularly known as prime number race [3].

Here we have compared the number of primes of form $4n + 1$ and $4n + 3$ for dominance till one trillion, i.e., 1,000,000,000,000 (10^{12}). The huge base data could be made available by using a just choice amongst the algorithms compared exhaustively in [5]. Java Programming Language, with its simple and lucid power highlighted in [7], was employed on an electronic computer to analyze prime range thoroughly.

Table 1: Number of Primes of form 4n + k In First Blocks of 10 Powers

Sr. No.	Range 1-*x* (1 to *x*)	Ten Power (*x*)	Number of Primes of the form $4n + 1$ $\pi_{4,1}(x)$	Number of Primes of the form $4n + 3$ $\pi_{4,3}(x)$
1.	1-10	10^1	1	2
2.	1-100	10^2	11	13
3.	1-1,000	10^3	80	87
4.	1-10,000	10^4	609	619
5.	1-100,000	10^5	4,783	4,808
6.	1-1,000,000	10^6	39,175	39,322
7.	1-10,000,000	10^7	332,180	332,398
8.	1-100,000,000	10^8	2,880,504	2,880,950
9.	1-1,000,000,000	10^9	25,423,491	25,424,042
10.	1-10,000,000,000	10^{10}	227,523,275	227,529,235
11.	1-100,000,000,000	10^{11}	2,059,020,280	2,059,034,532
12.	1-1,000,000,000,000	10^{12}	18,803,924,340	18,803,987,677

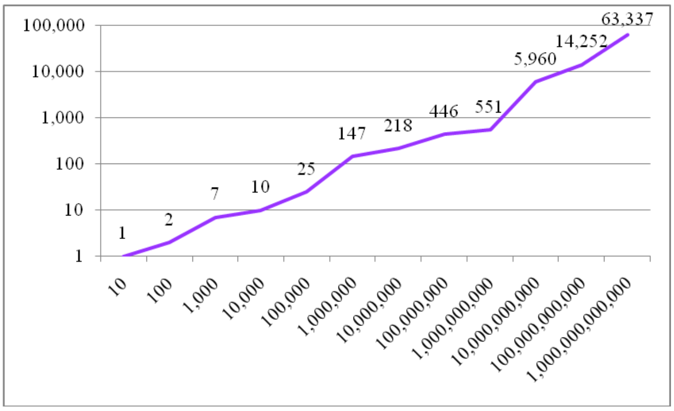


Figure 1: Dominance of $\pi_{4,3}(x)$ over $\pi_{4,1}(x)$

It is observed that the number of primes of the form $4n + 3$ is more than those of form $4n + 1$ in the initial block-wise ranges up to 10^{12}. Whether this trend of $\pi_{4,3}(x) > \pi_{4,1}(x)$ is all uniform and continues ahead on majority was a subject matter of interest for long and now has been settled. The first reversal has already happened in-between at $x = 26{,}861$ where $\pi_{4,3}(x) < \pi_{4,1}(x)$ [2] and is not visible here as we have taken only discrete values at 10 power blocks. But on majority it is $\pi_{4,3}(x)$ that is found to lead. Still interestingly, Littlewood [4] could show that this prime race switches back and forth infinitely many times.

Block-Wise Distribution of Primes

Neither is there a formula to consider all primes in simple go, nor are the primes finite in number to consider them all together. So, to understand their random-looking distribution, we have adopted a plain approach of considering all primes up to a certain limit, viz., one trillion (10^{12}) and dividing this complete number range under consideration in blocks of powers of 10 each :

1-10, 11-20, 21-30, 31-40, · · ·

1-100, 101-200, 201-300, 301-400, · · ·

1-1000, 1001-2000, 2001-3000, 3001-4000, · · ·

⋮

A rigorous analysis has been performed on many fronts. Since our range is 1-10^{12}, it is clear that there are 10^{12-i} number of blocks of 10^i size for each $1 \le i \le 12$.

The First and the Last Primes of form 4n + K in the First Block of 10 Powers

The inquiry is done of the first and the last prime in each first block of 10 powers till the range of 10^{12} under consideration. Of particular interest are the last primes, as the first primes of first power of 10 will naturally continue for all blocks ahead.

Table 2: First and Last Primes of form 4n + K In First Block of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	First Prime in the First Block		Last Prime in the First Block	
		Form $4n + 1$	Form $4n + 3$	Form $4n + 1$	Form $4n + 3$
1.	10	5	3	5	7
2.	100	5	3	97	83
3.	1,000	5	3	997	991
4.	10,000	5	3	9,973	9,967
5.	100,000	5	3	99,989	99,991
6.	1,000,000	5	3	999,961	999,983
7.	10,000,000	5	3	9,999,973	9,999,991
8.	100,000,000	5	3	99,999,989	99,999,971
9.	1,000,000,000	5	3	999,999,937	999,999,883
10.	10,000,000,000	5	3	9,999,999,929	9,999,999,967
11.	100,000,000,000	5	3	99,999,999,977	99,999,999,947
12.	1,000,000,000,000	5	3	999,999,999,989	999,999,999,959

While the first primes in all the first blocks have respective fixed values, the difference in the last primes of form $4n + 1$ and $4n + 3$ in the first blocks have zigzag trend.

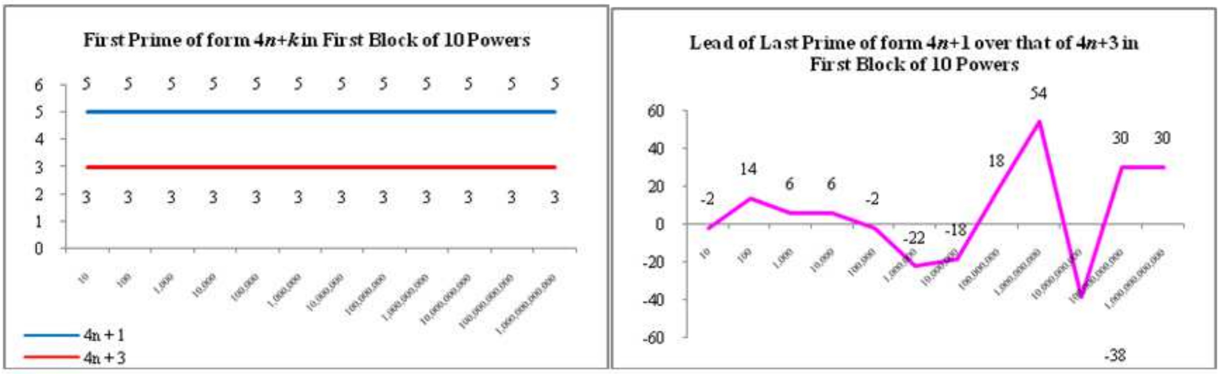


Figure 2: The First and the Last Primes of form 4n + k in the First Block of 10 Powers

MINIMUM NUMBER OF PRIMES OF FORM 4n + k IN BLOCKS OF 10 POWERS

Inspecting all blocks of each 10 power ranging from 10^1 to 10^{12} till 10^{12}, the minimum number of primes found in each 10 power block has been determined rigorously for primes of both forms under consideration.

Table 3: Minimum Number of Primes of form 4n + k in Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Minimum No. of Primes of form 4*n* + 1 in Block	Minimum No. of Primes of form 4*n* + 3 in Block
1.	10	0	0
2.	100	0	0
3.	1,000	1	1
4.	10,000	124	126
5.	100,000	1,655	1,642
6.	1,000,000	17,752	17,693
7.	10,000,000	180,103	180,155
8.	100,000,000	1,807,816	1,808,155
9.	1,000,000,000	18,094,582	18,096,669
10.	10,000,000,000	180,986,422	180,990,999
11.	100,000,000,000	1,812,949,220	1,812,975,212
12.	1,000,000,000,000	18,803,924,340	18,803,987,677

There is slight fluctuation in difference in minimum number of primes of form 4*n* + 1 and 4*n* + 3 in these blocks, but overall those of form 4*n* + 1 show a lead as far as minimality is considered.

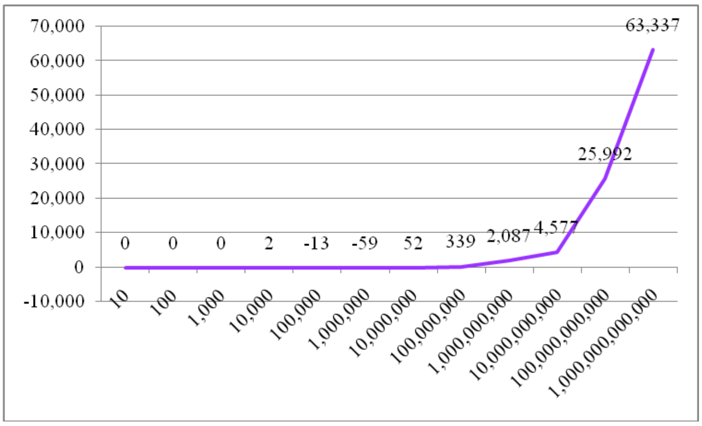


Figure 3: Minimality Lead of Number of Primes of form 4n+1 over 4n+3 in 10 Power Blocks

The first and last blocks in our range of one trillion with minimum number of primes of forms 4*n* + 1 and 4*n* + 3 in them are also determined.

Table 4 : First and Last 10 Power Blocks with Minimum Number of Primes of form 4n + k

Sr. No.	Blocks of Size (of 10 Power)	First Block with Minimum Number of Primes		Last Block with Minimum Number of Primes	
		Form 4*n* + 1	Form 4*n* + 3	Form 4*n* + 1	Form 4*n* + 3
1.	10	120	90	999,999,999,990	999,999,999,990
2.	100	39,600	31,400	999,999,999,700	999,999,999,400
3.	1,000	239,179,190,000	132,114,810,000	848,965,391,000	900,070,527,000
4.	10,000	886,052,870,000	850,297,060,000	886,052,870,000	947,512,030,000
5.	100,000	942,988,000,000	974,597,900,000	942,988,000,000	974,597,900,000
6.	1,000,000	999,835,000,000	950,269,000,000	999,835,000,000	950,269,000,000
7.	10,000,000	997,070,000,000	978,750,000,000	997,070,000,000	978,750,000,000
8.	100,000,000	997,500,000,000	991,800,000,000	997,500,000,000	991,800,000,000
9.	1,000,000,000	997,000,000,000	999,000,000,000	997,000,000,000	999,000,000,000
10.	10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
11.	100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

The comparative trend deserves graphical representation.

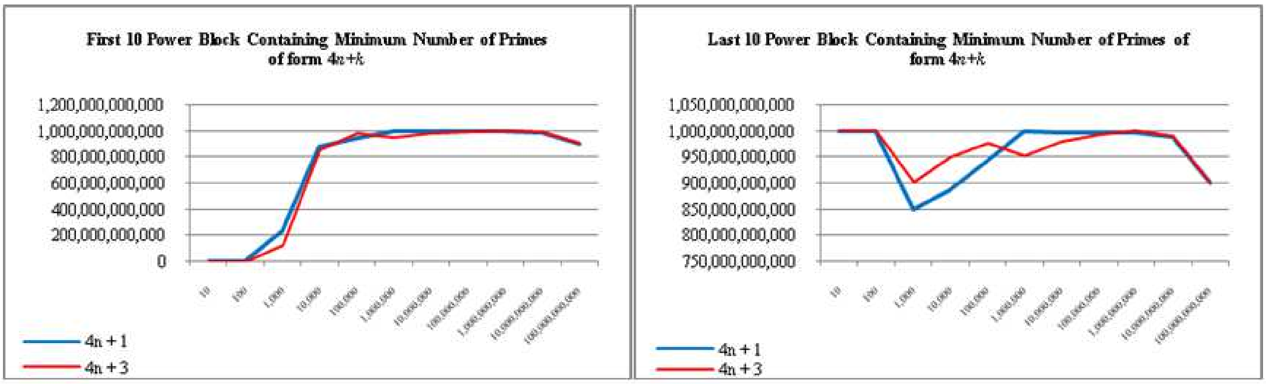


Figure 4 : First and Last 10 Power Blocks with Minimum Number of Primes of form 4n + k

Determination of frequency of minimum block spacing occurrence of primes of form $4n + k$ becomes due.

Table 5: Frequency of Minimum Number of Primes of Form 4n + k In Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Minimum Number of Primes of form $4n + 1$ Occurring in Blocks	Number of Times the Minimum Number of Primes of form $4n + 3$ Occurring in Blocks
1	10	81,819,591,486	81,819,537,592
2	100	1,224,999,973	1,225,056,750
3	1,000	9	6
4	10,000	1	2

For rest 10 powers blocks for both forms of primes, the number of blocks containing minimum number of primes become 1 and give following pattern for their percentage decrease.

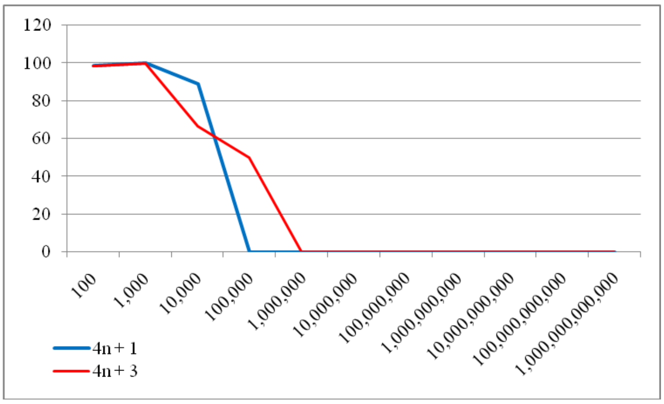


Figure 5: % Decrease in Occurrences of Minimum Number of Primes of form 4n + k in Blocks of 10 Powers

MAXIMUM NUMBER OF PRIMES OF FORM 4n + k IN BLOCKS OF 10 POWERS

All blocks of each 10 power ranging from 10^1 to 10^{12} till 10^{12} have also been analyzed for the maximum number of primes found in each of them.

Table 6: Maximum Number of Primes of form 4n + k in Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Maximum No. of Primes of form 4*n* + 1 in Block	Maximum No. of Primes of form 4*n* + 3 in Block
1.	10	2	2
2.	100	11	13
3.	1,000	80	87
4.	10,000	609	619
5.	100,000	4,783	4,808
6.	1,000,000	39,175	39,322
7.	10,000,000	332,180	332,398
8.	100,000,000	2,880,504	2,880,950
9.	1,000,000,000	25,423,491	25,424,042
10	10,000,000,000	227,523,275	227,529,235
11	100,000,000,000	2,059,020,280	2,059,034,532
12	1,000,000,000,000	18,803,924,340	18,803,987,677

Here primes of form 4*n* + 3 dictate in all blocks except the first block size of 10 where there is equality.

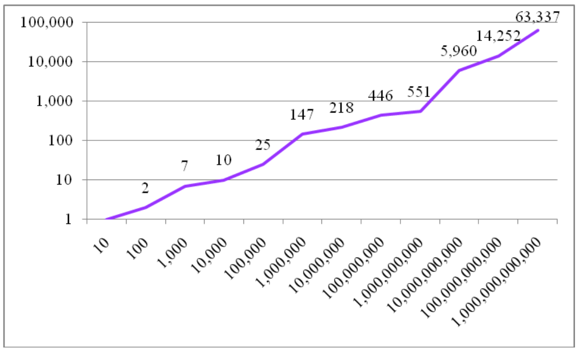


Figure 6: Maximality Lead of Number of Primes of form 4n+3 over 4n+1 in 10 Power Blocks

The first and last blocks in our range of one trillion with maximum number of primes of forms 4*n* + 1 and 4*n* + 3 in them are found to be:

Table 7: First and Last 10 Power Blocks with Maximum Number of Primes of Form 4n + k

Sr. No.	Blocks of Size (of 10 Power)	First Block with Maximum Number of Primes		Last Block with Maximum Number of Primes	
		Form 4*n* + 1	Form 4*n* + 3	Form 4*n* + 1	Form 4*n* + 3
1.	10	10	0	999,999,998,440	999,999,998,860
2.	100	0	0	935,419,728,400	0
3.	1,000	0	0	0	0
4.	10,000	0	0	0	0
5.	100,000	0	0	0	0
6.	1,000,000	0	0	0	0
7.	10,000,000	0	0	0	0
8.	100,000,000	0	0	0	0
9.	1,000,000,000	0	0	0	0
10.	10,000,000,000	0	0	0	0
11.	100,000,000,000	0	0	0	0

Since in general, the prime density shows a decreasing trend with higher range of numbers, it is natural that for larger block sizes, the first as well as the last occurrences of maximum number of primes in them starts in the block after 0, i.e., the very first block.

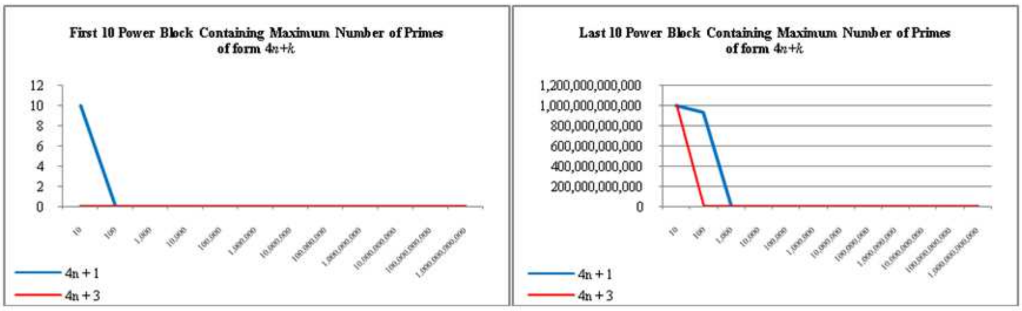


Figure 7: First and Last 10 Power Blocks with Maximum Number of Primes of form 4n + k

Decrease in the prime density asserts that the maximum number of primes cannot occur frequently, at least for higher ranges.

Table 8: Frequency of Maximum Number of Primes of form 4n + k In Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Maximum Number of Primes of form 4*n* + 1 Occurring in Blocks	Number of Times the Maximum Number of Primes of form 4*n* + 3 Occurring in Blocks
1.	10	623, 515, 826	623, 525, 269
2.	100	30	1
3.	1,000	1	1
4.	10,000	1	1
5.	100,000	1	1
6.	1,000,000	1	1
7.	10,000,000	1	1
8.	100,000,000	1	1
9.	1,000,000,000	1	1
10.	10,000,000,000	1	1
11.	100,000,000,000	1	1
12.	1,000,000,000,000	1	1

Here too, initially frequency of maximum primes of form 4*n* + 3 has surpassed that of form 4*n* + 1, then 4*n* + 1 has taken a marginal lead over earlier and the figure for both has settled down to 1.

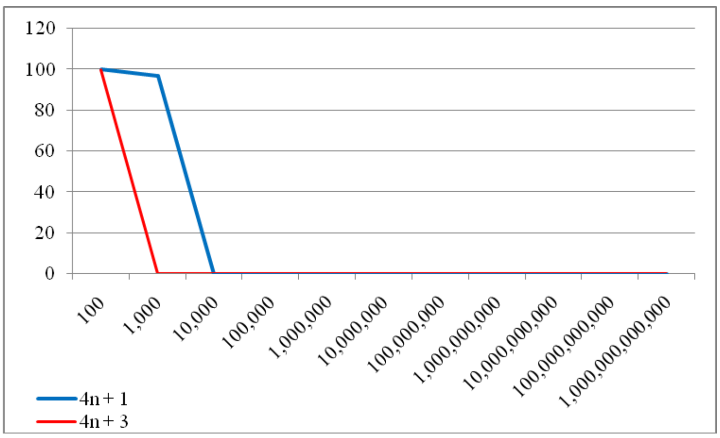


Figure 8: % Decrease in Occurrences of Maximum Number of Primes of form 4n+k in Blocks of 10 Powers

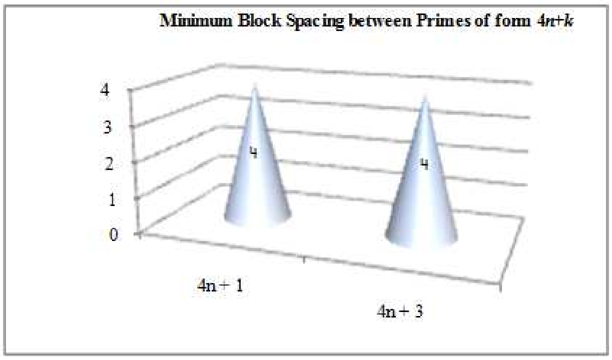
BLOCK-WISE DISTRIBUTION OF PRIMES

To analyze the distribution of the primes, it is worthwhile to inspect minimum spacings between successive primes.

Minimum Spacing between Primes of FORM 4n + K IN BLOCKS OF 10 POWERS

Exempting prime-empty blocks, the minimum spacing between primes of form 4*n* + 1 and 4*n* + 3 in blocks of 10 powers is determined to be 4 each, beginning with the first power block 10^1 = 10. Since for larger block sizes, the minimum spacing value cannot increase, it remains same ahead for all blocks of all higher powers of 10 in all ranges, even beyond our range of a trillion, virtually till infinity!

The minimum block spacing of 4 for primes of form 4*n* + 1 occurs last in our range at 999,999,997,753 for blocks of 10 and at 999,999,998,509 for all higher blocks. While the same for primes of form 4*n* + 3 occurs last at 999,999,998,863 for all blocks.



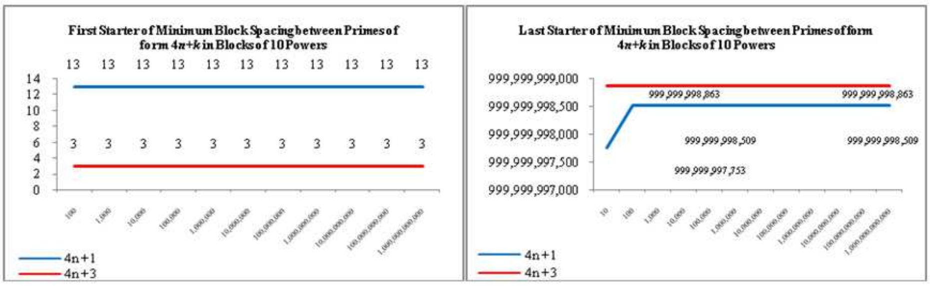


Figure 9: First & Last Starters of Minimum Block Spacing between Primes of form 4n+k in Blocks of 10 Powers

It is worthwhile to determine how many times this minimum block spacing occurs between primes of form 4*n* + 1 and 4*n* + 3. The following table shows this data:

Table 9: Number of Times the Minimum Block Spacing Occurring for Primes of form 4n + k

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Minimum Block Spacing Occurring for Primes of form 4*n* + 1	Number of Times the Minimum Block Spacing Occurring for Primes of form 4*n* + 3
1.	10	311,753,844	311,762,851
2.	100	872,913,857	872,957,565
3.	1,000	929,032,068	929,087,503
4.	10,000	934,643,802	934,695,345
5.	100,000	935,203,844	935,256,934

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Minimum Block Spacing Occurring for Primes of form 4*n* + 1	Number of Times the Minimum Block Spacing Occurring for Primes of form 4*n* + 3
Table 9 : Contd.,			
6.	1,000,000	935,260,090	935,313,036
7.	10,000,000	935,265,678	935,318,558
8.	100,000,000	935,266,208	935,319,139
9.	1,000,000,000	935,266,256	935,319,191
10.	10,000,000,000	935,266,264	935,319,194
11.	100,000,000,000	935,266,265	935,319,194
12.	1,000,000,000,000	935,266,265	935,319,194

There is increase in the number of times the minimum spacing occurs for primes of both forms. This is because whenever we increase block size, some primes with desired spacing occurring at the crossing of earlier blocks find themselves in same larger blocks raising the count. Of course, this rate of increase gradually decreases for primes of both forms, as shown in graph, as we reach the block size of our limit. In graph, we have dropped the first increase as it is exceptionally tremendous as much as 180%.

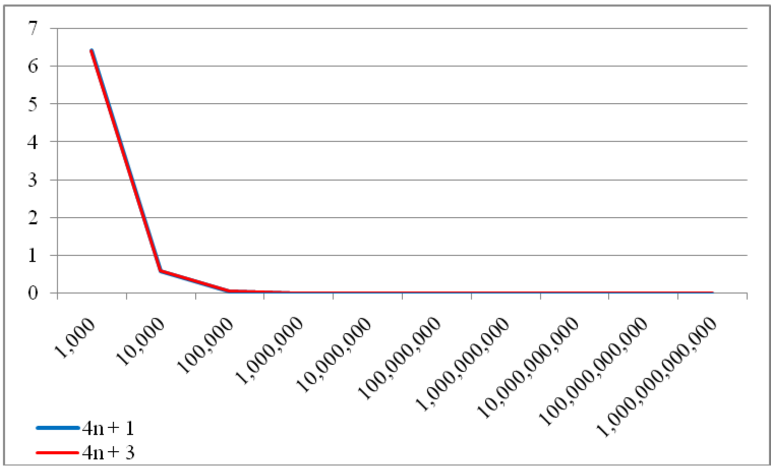


Figure 10: % Increase in Occurrences of Minimum Block Spacing between Primes of form 4n+k in Blocks of 10 Powers

MAXIMUM SPACING BETWEEN PRIMES OF FORM 4n + k IN BLOCKS OF 10 POWERS

Unlike the minimum spacing between primes in blocks of 10 powers, the maximum spacing in these blocks goes on increasing with increase in the block size. Till our ceiling of one trillion, the following trend of increase and settling is seen.

Table 10: Maximum Block Spacing Occurring for Primes of form 4n + k

Sr. No.	Blocks of Size (of 10 Power)	Maximum Block Spacing Occurring for Primes of form 4*n* + 1	Maximum Block Spacing Occurring for Primes of form 4*n* + 3
1.	10	8	8
2.	100	96	96
3.	1,000	920	948
4.	10,000	1,084	1,056
5.	100,000	1,084	1,056
6.	1,000,000	1,084	1,056

Sr. No.	Blocks of Size (of 10 Power)	Maximum Block Spacing Occurring for Primes of form 4*n* + 1	Maximum Block Spacing Occurring for Primes of form 4*n* + 3
Table 10 : Contd.,			
7.	10,000,000	1,084	1,056
8.	100,000,000	1,084	1,056
9.	1,000,000,000	1,084	1,056
10.	10,000,000,000	1,084	1,056
11.	100,000,000,000	1,084	1,056
12.	1,000,000,000,000	1,084	1,056

In our range of inspection of 1 trillion, except for the blocks of 10, 100 & 1000; where there is a match in former 2 and reverse lead in the later; in-block maximum spacing for primes of form 4*n* + 1 is more than the other.

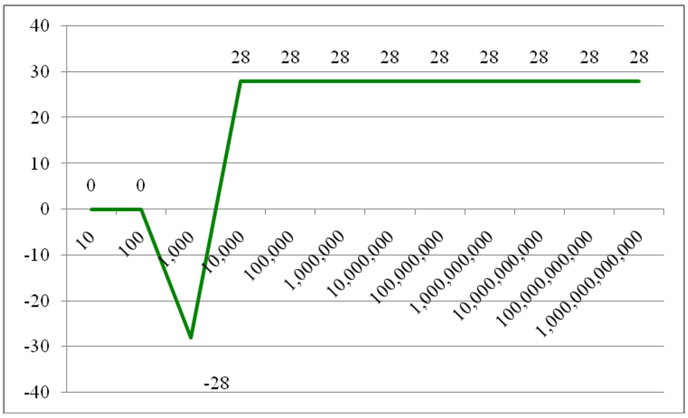


Figure 11: Dominance of Maximum Block Spacing between Primes of form 4n+1 over 4n+3

The first and last primes of forms 4*n* + 1 and 4*n* + 3 with these maximum in-block spacings for various blocks are as follows.

Table 11: First and Last Primes of Form 4n + k with Maximum Block Spacings

Sr. No.	Blocks of Size (of 10 Power)	First Prime with Respective Maximum Block Spacing		Last Prime with Respective Maximum Block Spacing	
		Form 4*n* + 1	Form 4*n* + 3	Form 4*n* + 1	Form 4*n* + 3
1.	10	101	11	999,999,998,441	999,999,993,071
2.	100	235,901	257,003	999,999,916,901	999,999,867,203
3.	1,000	584,989,617,053	797,455,218,031	584,989,617,053	797,455,218,031
4.	10,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
5.	100,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
6.	1,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
7.	10,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
8.	100,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
9.	1,000,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
10.	10,000,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
11.	100,000,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347
12.	1,000,000,000,000	769,627,303,309	635,666,324,347	769,627,303,309	635,666,324,347

The comparative trend is clear from graphical representation.

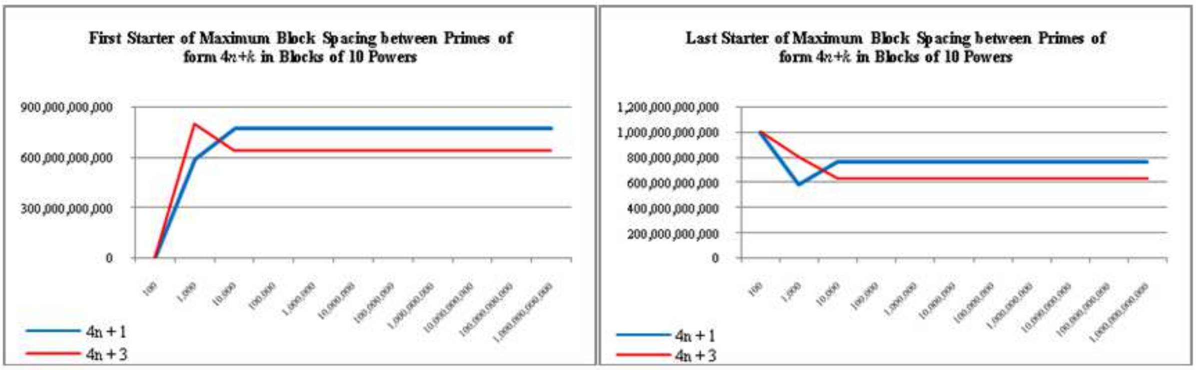


Figure 12 : First & Last Starters of Maximum Block Spacing between Primes of form 4n+k in Blocks of 10 Powers

Determination of frequency of maximum block spacing occurrence of primes of form 4*n* + *k* is done.

Table 12: Frequency of Maximum Block Spacing Occurrence of Primes of form 4n + k

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Maximum Block Spacing Occurs for Primes of form 4*n* + 1	Number of Times the Maximum Block Spacing Occurs for Primes of form 4*n* + 3
1.	10	311,761,982	311,762,418
2.	100	22,195,113	22,196,682

For all rest 10 powers blocks for both prime forms, the maximum block spacing occurs only once yielding following pattern for their respective differences – initially frequency of 4*n* + 3 type maximum spacings dominating followed by no difference.

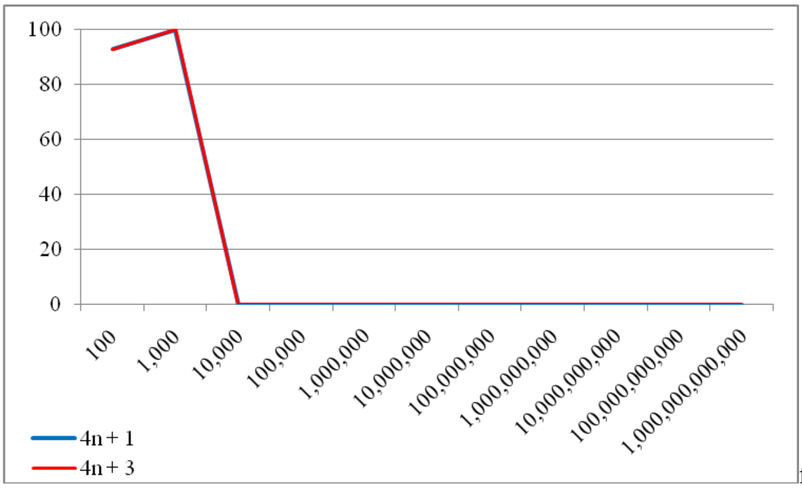


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Figure 13: % Decrease in Occurrences of Maximum Block Spacing between Primes of form 4n+k in Blocks of 10 Powers

END DIGITS OF PRIMES

UNITS PLACE DIGITS IN PRIMES

Prime numbers can have only six different possible digits in units place. Exhaustive analysis revealed the number of primes for form $4n + k$ with different digits in units place to be as follows:

Table 13: Number of Primes of form 4n + K with Different Units Place Digits

Sr. No.	Digit in Units Place	Number of Primes of Form	
		$4n + 1$	$4n + 3$
1.	1	4,700,974,794	4,700,986,186
2.	2	0	0
3.	3	4,700,990,890	4,700,989,014
4.	5	1	0
5.	7	4,700,987,961	4,701,009,039
6.	9	4,700,970,694	4,701,003,438

2 is only even prime and 5 in only prime with its unit place digit. So analysis ahead has neglected 2 and 5 in units places as they have exceptional nature.

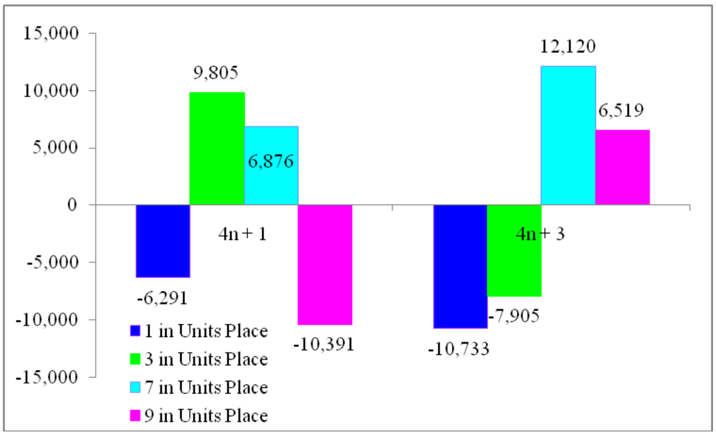


Figure 14: Deviation of Unit Place Digits of Primes of form 4n+k from Average

TENS AND UNITS PLACE DIGITS IN PRIMES

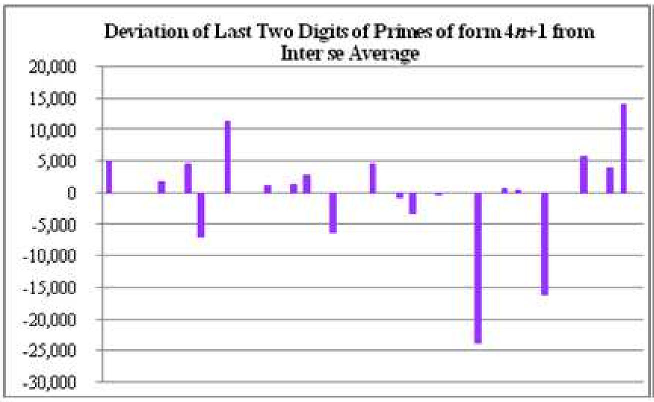
There are 42 numbers possible in combination of tens and units place of any prime number. The number of primes with these is also determined in range under consideration.

Table 14

Sr. No.	Digits in Tens & Units Place	Number of Primes of Form	
		$4n + 1$	$4n + 3$
1.	01	940,201,224	0
2.	02	0	0
3.	03	0	940,199,042
4.	05	1	0
5.	07	0	940,201,524
6.	09	940,198,037	0
7.	11	0	940,191,631
8.	13	940,200,704	0

Sr. No.	Digits in Tens & Units Place	Number of Primes of Form	
		4*n* + 1	4*n* + 3
Table 14: Contd.,			
9.	17	940,189,305	0
10.	19	0	940,224,567
11.	21	940,207,451	0
12.	23	0	940,205,113
13.	27	0	940,207,372
14.	29	940,197,429	0
15.	31	0	940,201,296
16.	33	940,197,634	0
17.	37	940,198,836	0
18.	39	0	940,195,363
19.	41	940,190,006	0
20.	43	0	940,197,593
21.	47	0	940,197,732
22.	49	940,200,776	0
23.	51	0	940,204,880
24.	53	940,195,587	0
25.	57	940,192,995	0
26.	59	0	940,199,522
27.	61	940,196,110	0
28.	63	0	940,195,366
29.	67	0	940,203,357
30.	69	940,172,444	0
31.	71	0	940,196,489
32.	73	940,196,947	0
33.	77	940,196,643	0
34.	79	0	940,188,826
35.	81	940,180,003	0
36.	83	0	940,191,900
37.	87	0	940,199,054
38.	89	940,202,008	0
39.	91	0	940,191,890
40.	93	940,200,018	0
41.	97	940,210,182	0
42.	99	0	940,195,160

Neglecting the cases 02 and 05 as well as those where there are no occurrences, following deviation from average is seen for occurrence of other possibilities of last two digits in range of 1-10^{12} for primes of each form 4*n* + *k*.



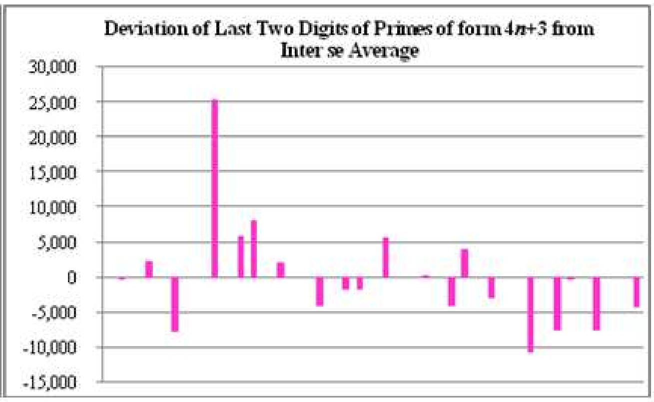


Figure 15: Deviation of Last Two Digits of Primes of form 4n+k from Inter se Average

ANALYSIS OF SUCCESSIVE PRIMES OF FORM 4*n* + *k*

The case when two successive primes are of same form; either $4n + 1$ or $4n + 3$; is interesting. The number of successive primes of desired forms is as follows.

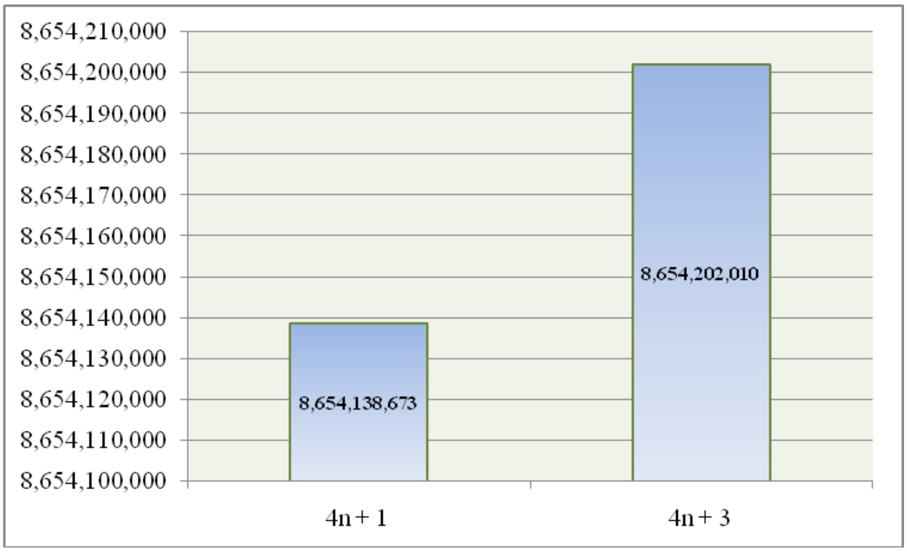


Figure 16 : Number of Successive Primes of form 4n + $k \leq 10^{12}$.

We have exhaustively analyzed the cases of successive primes of specific forms. The minimum spacing between successive primes of forms $4n + k$ has following properties.

Figure 17

The maximum spacing between successive primes of forms $4n + k$ has following properties.

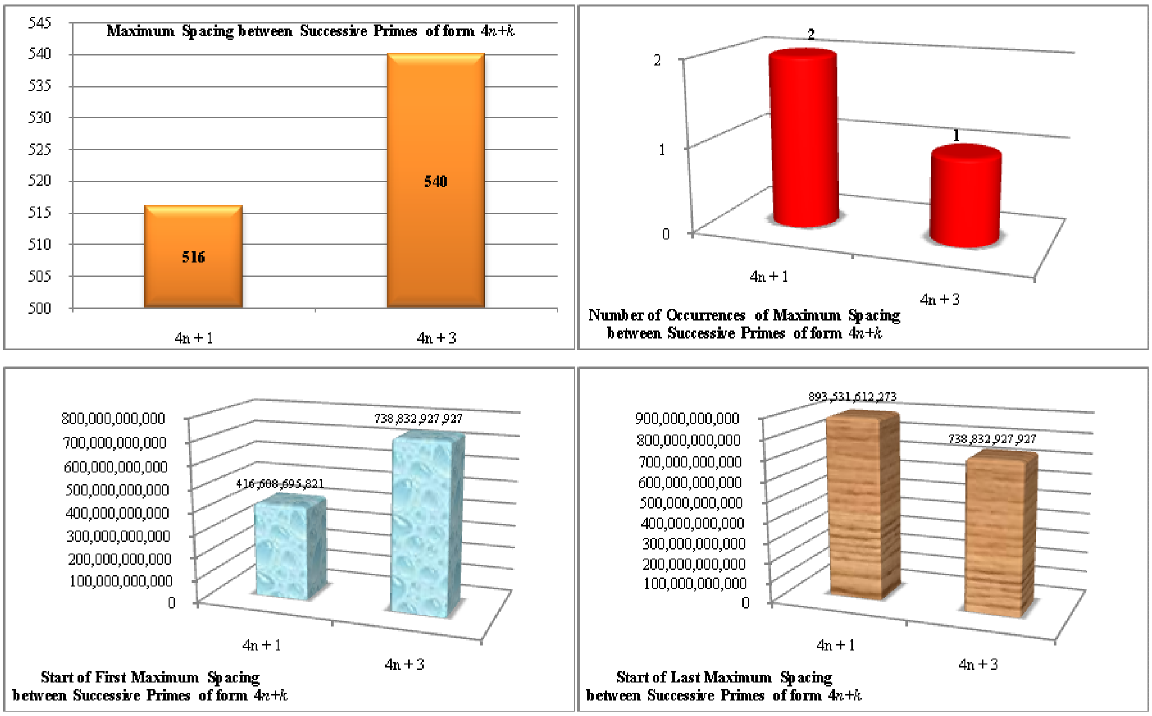


Figure 18

There have been consistent efforts to study random distribution of primes. Distribution patterns of primes of form $3n + k$ have been recently analyzed [6]. The work presented here is an addition to that with respect to a specific linear pattern of $4n + k$. The author is sure that the availability of rigorous analysis like this will help give a deeper insight into prime distribution.

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